MPOWER!

FOR KIDS WHO LIKE TO SOLVE PROBLEMS

CONGRATULATIONS TO OUR GIRLS IN MATH AT **YALE WINNERS!**

RSM
Russian School of Mathematics

CONTENT

Toys and Games, #8, 2024

NOTE FROM THE EDITOR

There is no such thing as a "math person." However, a child's early learning experiences directly impact his perception of his mathematics ability. When children are taught mathematics through fun and engaging problems, they not only build fluency in and an intuitive understanding of mathematics, but they also develop critical problem-solving skills.

This issue of MPower! focuses on toys and games. Whether you play chess or dominoes, roll dice, or arrange tiles on a board, you can explore the game's underlying logic and strategy. Both game theory and probability theory originated with the study of games. Now these mathematics are used in physics and biology, in finance and economics, in political science and AI. Which game will lead you to a wonderful math discovery?

We hope you enjoy the stories and problems inside.

KEY

●○○○ GRADES 1 & UP
●●○○ GRADES 3 & UP
●●●○ GRADES 5 & UP
●●●● MASTER SOLVER

QUESTIONS. COMMENTS. SUGGESTIONS

Connect with us at: MPower@russianschool.com

Congratulations!

GIRLS IN MATH AT YALE COMPETITION

In the Individual Round, RSM students **Selena Ge** and **Anika Mittal** secured

2nd & 3rd place

Team RSM placed 7th overall!

SELENA GE
4 Years with RSM
2nd in Individual Round

ANIKA MITTAL
8 Years with RSM
3rd in Individual Round

ALLI KATILA-MIIKKULAINEN
6 Years with RSM

CHELSEA YAN
6 Years with RSM

ERICA HO
9 Years with RSM

ANNA TANG
9 Years with RSM

MPOWER COMPETITION

RSM MCP is proud to present The winners of the MPOWER competition. Earning a place in the MCP branch is an achievement itself but winning the MPOWER contest proves you are the "best of the best" and is a super achievement that requires talent, dedication, and passion. The winners have demonstrated these traits in abundance.

MPower is an internal MCP competition.

NMCP WINNERS

GRADE 4

- Evelyn Sun
- Arthur Zhang
- Bhagat Olety
- Dhanish Kola
- Michael Ratiu
- Vivaan Soni

GRADE 5

- Anna Ruan
- Daniel Duan
- Michael Shi
- Nathan Bar
- Sanjay Malhotra
- Sophie Li

GRADE 6

- Caitlin Lin
- Harika Appalla
- Lucas Washow
- Melvin Ma
- Mia Long
- Ryan Xiao
- Steven Brody
- Taehee Ha

GRADE 7

- Harshika Murikoti
- Rebeca Costin
- Aarush Kommaraju
- Alexander Jovanovic
- Benjamin Ren
- Leo Abu-Ghoush
- Max Qiu
- Oliver Abu-Ghoush
- Ria Vartak
- Ruoyu Zhou
- Sasha Nguyen
- Tanvi Nasika

GRADE 8

- Simon Cheeren
- Aarush Joglekar
- Abe Narayan
- Alexander Amrhein
- David He
- Grace Zheng
- Max Jin
- Michael Xie
- Neil Iyer
- Samuel Minow
- Shivam Chowdhary

GRADE 9

- Alli Katila-Miikkulainen
- Camea Caprita
- Nikhil Byrapuram
- Timur Kilybayev
- Yury Kirpichev

GRADE 10

- Daniil Landau
- Ethan Chen
- Jianing (Jenny) Huang
- Luca Pieleanu
- Yury Bychkov

GRADE 11

- Arthur Li
- Brandon Ni
- Jefferson Ji
- Rhishi Sakthivel

MCP WINNERS

GRADE 4

Advait Anand	Bay Area MCP, CA
Agastya Mantha	Metrowest, MA
Ishaan Sharma	Metrowest, MA
Nate Yang	Online MCP
Owen Li	Online MCP
Aditya Dhamdhere	Bay Area MCP, CA
Alex Louie	Bay Area MCP, CA
Anirudh Rengarajan	Online MCP
Avik Saha	Bay Area MCP, CA
Beliz Tuna	Online MCP
Matthew Popescu	Online MCP

GRADE 5 Level 1

Aditya Karri	Online MCP
Charles Sokolov	Online MCP
Sharvil Jamdade	Online MCP
Amy Zhang	Online MCP
Daniel Zheng	Metrowest, MA
James Ho	Metrowest, MA
Mandy Liu	Metrowest, MA

GRADE 5 Level 2

Aarnav Daivik	Online MCP
Arjun Shenoy	Bay Area MCP, CA
Prasanna Vijay	Bay Area MCP, CA
Zere Nagashybek	Herndon, VA
Anaye Agrawal	Bay Area MCP, CA
Carol Ma	Online MCP
Erik Prokhorov	Online MCP
Eunice Shey	Online MCP
Ishan Sinha	Bay Area MCP, CA
Jonathan Cohen	Herndon, VA
Leo Silverman	Lexington, MA
Maria Pozhogina	Andover, MA
Rohan Hallur	Bay Area MCP, CA
Shubhit Jain	Online MCP

GRADE 6 Level 1

Gioia De Noue	Online MCP
Jayden Sun	Online MCP
Mona Hou	Online MCP
Pranav Kurupati	Online MCP
Virinchi Madireddy	Online MCP
Vivasvan Gupta	Online MCP

GRADE 6 Level 2

Alan Yu	Bay Area MCP, CA
Arjun Poola	Bay Area MCP, CA
Avani Daftuar	Online MCP
Emma Xu	Bay Area MCP, CA
Jessie Wang	Bay Area MCP, CA
Paarth Bansal	Bay Area MCP, CA
Riley Chin	Online MCP
Sofia Rosolovskaya	Online MCP
Steven Yang	Bay Area MCP, CA
Tiana Liu	Online MCP
Warren Chai	Online MCP

GRADE 7 Level 1

Caleb Wong	Online MCP
Daniel Mirgorodskiy	San Mateo, CA
Jonathan Gu	Andover, MA
Klim Zubkov	San Mateo, CA
Matthew Agapitov	Online MCP
Ryan Blocker	Online MCP
Shlok Khakhkhar	San Mateo, CA
Vincent Chan	South O. C., CA

GRADE 7 Level 2

Aarav Raj	Bay Area MCP, CA
Advik Jain	Bellevue, WA
Arielle Litosh	Herndon, VA
Artem Godlevskii	Herndon, VA
Chloe Chen	Bay Area MCP, CA
Eleanor Wang	Online MCP
Isabel Zheng	Bay Area MCP, CA
Kevin Li	Online MCP
Radha Agashe	Herndon, VA
Samanvay Srivatsa	Bay Area MCP, CA
Siddh Mehta	Online MCP

GRADE 8 Level 1

Aarav Karumanchi	Online MCP
Aarav Varshneya	Online MCP
Aaron Anghel	Online MCP
Akansh Karthik	Online MCP
Anshu Vakkantula	Online MCP
Cecilia Jia	Online MCP
Dylan Raman	Online MCP
Ellen Kandilov	Online MCP
Jasper Guan	Online MCP
Sanskriti Bukkapatnam	Online MCP

GRADE 8 Level 2

Martin Petrov	Herndon, VA
Maya Orlova	Herndon, VA
Annika Ye	Bay Area MCP, CA
Justin Nam	Herndon, VA
Justin Lee	Andover, MA
Katherine Gerasimov	Bellevue, WA
Klea Kukaj	UES 1 Ave, NY
Robert Agapie	South O.C., CA

GRADE 9

Chloe Luo	Bay Area MCP, CA
Dorsa Motaharinezhad	Herndon, VA
Parsa Motaharinezhad	Herndon, VA
Rithvik Muthuvelu	Online MCP
Shveta Sunkar	Herndon, VA
Stephan Biletsky	Herndon, VA

SCALIFY-MAGNIFY!

"Have you seen?!" asked Bella. "Mr. Stephenson sent us a steam train for Christmas!"

"A real one?" asked Bella's younger brother Jan.

"Absolutely! It's made of steel and has all the teensy wheels, mini tubes, and even the dainty whistle!"

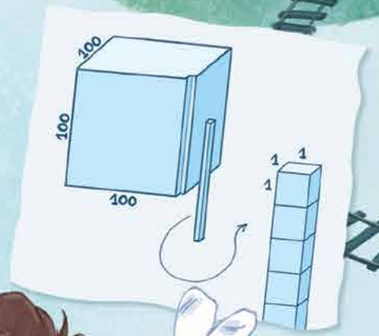
Jan pondered for a moment and then doubted his sister. "Teensy-tiny? Seriously? How can train wheels be tiny?"

"Ah," answered Bella, "well, of course it's a *toy* train at a scale of 1 : 64, but it's made exactly like a real one! It's called The Santa Fe Super Express!"

"Well," said Jan skeptically, "then it must be super heavy. If the real train is at least 100 tons, then ours is 100 divided by 64..., which is more than a ton, right?"

Bella was puzzled. The calculations were on target, but she could lift the model train by herself! How could that be?

Later, Bella was doing her homework and suddenly understood. Her homework problem asked, "How many cubic centimeters are in 1 cubic meter?" And the answer was not 100! It was 1,000,000! So – of course – her wonderful toy model weighs 64 × 64 × 64 = 262,144 times less, not just 64 times less than the real train!



The set of wooden cubes that Jan received as a Christmas gift contains 20 small cubes with side lengths of 1 inch and 3 large cubes with side lengths of 2 inches. Do 20 small cubes or 3 large cubes weigh more?

Jan decided to build a fire station and a red barn out of his wooden cubes, so he painted all of the small cubes red. He used 5 tubes of paint. How many tubes of paint will Jan need to paint all of his large cubes?

Bella suddenly remembered that she had been amazed by the perfect model of the Eiffel Tower at the Architecture and Construction Fair at school. It was made of thin steel wire and looked exactly like the real tower! The model was a bit taller than Bella! She had thought, "Wow, they probably need at least four kids to carry this tower."

Estimate the weight of the 5-foot high model if the real Eiffel Tower is 1,083 feet high and weighs 10,000 tons.

That night, Bella tried to do all the calculations in her head, and she got so tired that she fell asleep as soon as she started to read a new fantasy book.

Turn the page to learn what Bella saw in her dream!

Princess Elara is destined to marry a person who gives her a magic diamond. Sir Aldric the Modest only has 1 gold coin in his wallet, but he decides to try his luck. In the window of a magic jewelry store, he sees a huge diamond worth 1,000,000 gold coins. The jewelsmith tells Sir Aldric that the cost of a diamond is proportional to the square of its mass. If a diamond is twice as heavy as another diamond, then it is four times as expensive. And if a diamond is three times as heavy as another diamond, then it is nine times as expensive. How many times must the size of the diamond in the window be reduced for Sir Aldric to afford it?

The next morning, Jan was playing with his cat Loki, who was bravely hunting for Jan's stuffed deer. "You're just a scary tiger!" teased Jan, "But you're just a little too small." Loki did not answer, and instead prepared to grab a victim. "If only I had a magic wand," said Jan, "I would make you a tiger with a wave of my wand. What would be the name of the spell? Oh, I know! 'Scalify-magnify 4!' Oh, Loki, you are the most powerful tiger now! You are the size of a Bengal tiger, the biggest cat on the planet! Wait, no, no! It's not enough! 'Scalify-magnify 100!' Now you are 100 times as tall as a Bengal tiger! You are a Super Tiger! I will call you Centum-Tigris!"

"Hold on," said Bella. "It's not that simple! Do you think Centum-Tigris, a Bengal tiger, or Loki our house cat is more powerful?"

Centum means 100 in Latin. Centimeter, Century, Centipede, Centime, Centurion

Which cat sees better in the dark?

Light enters the eye through the pupil, so an animal whose pupil is larger in *area* can see better in the dark.

Which cat jumps higher and runs faster?

The cross-sectional *area* of its muscles is one of the factors that determines an animal's strength.

How much heavier is Centum-Tigris and a Bengal tiger than Loki the house cat?

The mass of an animal depends on the *volume* of its body.

Which cat freezes faster in cold weather?

The amount of heat that an animal's body loses every second is proportional to its *surface area*.

Which cat cools faster in hot weather?

Jan was puzzled. "Hmm... I have always wondered why elephants and hippos have strong legs but cannot jump. And frogs and grasshoppers have thin legs, but they are the best jumpers in the animal world! So, would my Centum-Tigris not be able to jump either? Why wouldn't he?"

"I think I know why! This is all because volume grows like a power of 3 while area grows like a power of 2," said Bella.

"Hmmm.... This sounds complicated. Is it like our model train?"

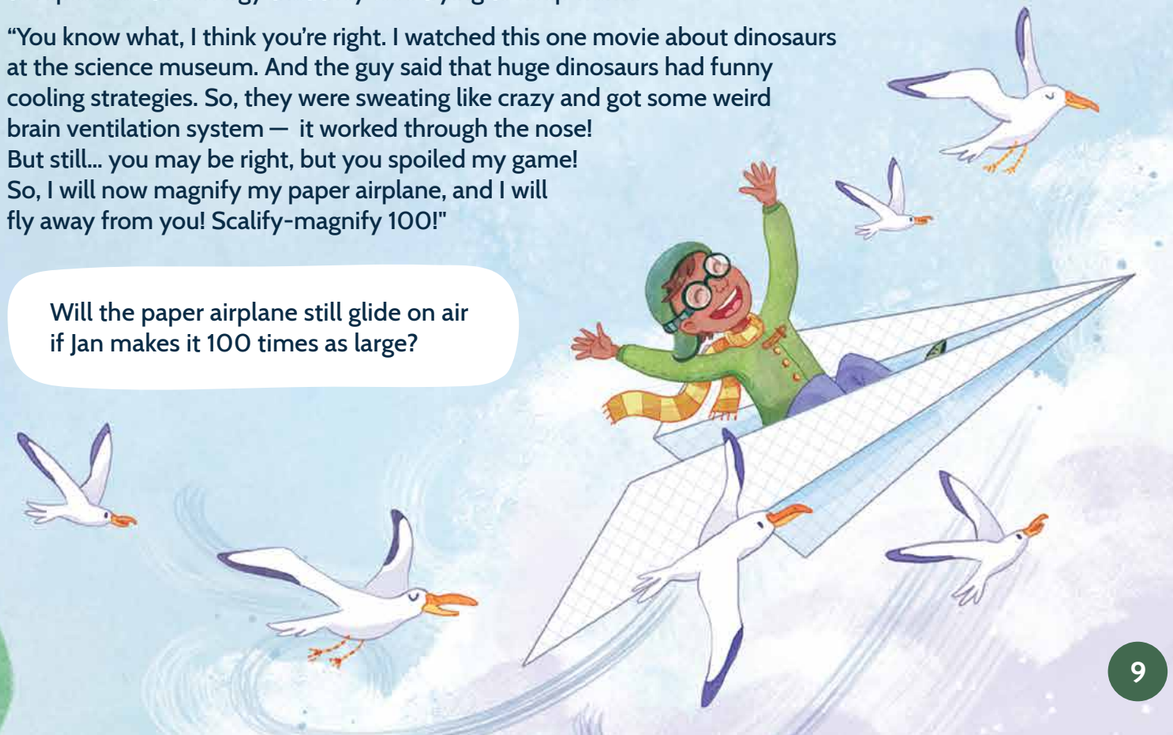
"Yes, exactly! And I think I figured out why elephants have huge ears and skin wrinkles — they help with overheating! Heat escapes through all the extra surface area and cools them off!"

Jan thought for some time. He was skeptical. "So, if that's true, tiny animals would have the opposite problem. They would cool off way too quickly. How could they fight that? They don't have heaters in their homes like we do. Ah-ha!"

"Oh, it's actually simple. I read about it in the children's encyclopedia! Small animals have to eat a lot to replace all the energy that they lose trying to keep warm."

"You know what, I think you're right. I watched this one movie about dinosaurs at the science museum. And the guy said that huge dinosaurs had funny cooling strategies. So, they were sweating like crazy and got some weird brain ventilation system — it worked through the nose!
But still... you may be right, but you spoiled my game!
So, I will now magnify my paper airplane, and I will fly away from you! Scalify-magnify 100!"

Will the paper airplane still glide on air if Jan makes it 100 times as large?



MISSING PIECES

MCHALLENGE

1

●○○○

Jake has seven cards with the digits 0, 1, 3, 4, 6, 7, and 8. He puts all seven cards in a row and gets a seven-digit number. What is the least possible odd number Jake can get?

MCP

2

●●○○

Stephen has three spinners, which are shown below. He spins the first spinner and gets a digit. If the digit is odd, he spins the second spinner, but if the digit is even, he spins the third spinner instead. For each of the two spins, Stephen writes down the digit he gets, so he ends up with a two-digit number in which the tens digit is from the first spinner. How many different two-digit numbers can Stephen get?

MCP

3

●●●●

Mom has two identical chocolate bars and five identical cookies. She wants to give the treats to Anita, Brian, and Chloe. In how many ways can she do it?

NMCP

4

●●○○

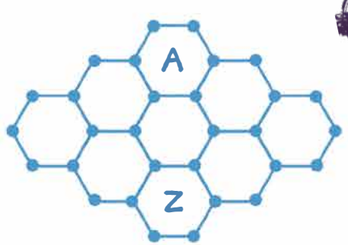
Some RSM 4th graders, all of whom were born in the same year, signed up for an "Invent a board game" contest. Ms. Clever said, "I don't know any of the students yet. But I know for sure that at least 6 of them were born on the same day of the week. However, I do *not* know for sure whether at least 4 of the students were born in the same month." How many students signed up for the contest?

MCP

5

●●●○

Billy the Bee sits in cell A of the honeycomb shown below. With one move, he can go to any adjacent cell that's relatively lower than his current cell. In how many ways can Billy get to cell Z?



MCP

6

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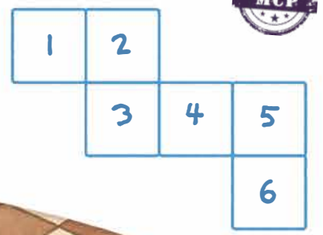
Jane cut out the net shown below and folded it into a cube. What number is opposite the side labeled 6?



MCP

7

●●●●

Audrey has a rectangular piece of cardboard with a length of 54 inches and a width of 21 inches. She starts to cut the cardboard into pieces for a new board game that she invented. With each cut, Audrey cuts off the largest possible square and continues until she only has squares left. How many squares does she have?

NMCP

8

●●○○

The Games Club meets all seven days of the week but only plays "Snacks and Splatters" on Mondays, Wednesdays, and Thursdays. Sophia wants to go to Games Club for 9 days in a row, and she wants to play "Snacks and Splatters" on as many days as possible. What day of the week should Sophia start going to the club?

MCP

BEAT THE OFFBEAT OGRE

We have no choice – we need to defeat this monster! The Offbeat Ogre is very dangerous.

It sings out of tune non-stop, and nobody can stand it for more than 1 minute!

See, all our friends are already down! But one good thing is that Ogre's singing repelled all the magic mosquitoes! Roll your dice!

But which die should I roll? I have 7 of them! I am so confused! I just know that I need to have at least 7 points of damage to beat Ogre!

First you need to decide which spell to cast. Which spells does your character know? What's his name again?

Tartoonist! He can bake and play music, and he knows only 2 spells! My character sheet says that the Cupcake spell does 1d12 silencing damage, and the Pillow spell does 2d6 silencing damage. Both should work, but I have no idea what 1d12 and 2d6 mean!

This is easy! For 1d12, you roll your 12-sided die once, and for 2d6, you roll your 6-sided die twice. And then you add the numbers you rolled.

1d12 =

2d6 =

My 12-sided die once or my 6-sided die twice? But is there any difference? Which should I choose if I want the highest number possible?

Game of Trust

Did you know that some mathematicians study games? In fact, game theory is a special branch of mathematics that focuses on optimal behavior in games. The games that mathematicians study are different from what we play with our friends, but similar games are all around us. Let's explore a famous thought experiment called...

THE PRISONER'S DILEMMA

Two people are accused of committing a crime together and are held in separate prisons. Each must decide whether to keep silent or to confess and betray the other. If both prisoners stay silent, each will be sentenced to 1 year in prison. If one of the prisoners confesses and betrays the other while the second prisoner remains silent, the betrayer will go free, while the silent prisoner will serve 10 years in prison. However, if *both* confess and betray the other, each will serve 5 years in prison. What should the prisoners do if each is concerned only with his own sentence?

Hold on! Is this a game? Mathematically speaking, yes, it is. In game theory, a game is any situation in which all the participants make choices, and what happens next depends on the choices made.

Let's organize the prisoner's game into a payoff table that shows all four of the possible outcomes of the game.

The first prisoner thinks, "I don't know for sure what he will do. If he betrays me, it'd be better to betray him, and serve 5 years instead of 10. On the other hand, if he stays silent, it'd be better to betray him and go free. So, whatever he does, I always do better if I betray him."

The second prisoner follows the same logic. So, it is highly likely that each will betray the other and get a 5 year sentence.

		2nd prisoner's choice: Keep silent	2nd prisoner's choice: Betray
1st prisoner's choice	Keep silent	-1, -1	-10, 0
1st prisoner's choice	Betray	0, -10	-5, -5

Could each prisoner serve only 1 year? Yes, but that outcome is unlikely since it relies on mutual trust: the only way either prisoner would stay silent is if he believes the other will do the same. A lack of mutual trust is a key feature in this game!

But there is a paradox in the Prisoner's Dilemma: the best outcome seems unreachable! Since both players have initially chosen to betray, if only one prisoner then changes his strategy, he will get more years in prison than he has now. This often happens in games — if you change your initial strategy and the other player does not, you often become worse off than you were.

John Forbes Nash Jr. (1928-2015) was an outstanding mathematician. He discovered the possibility of a non-cooperative stable equilibrium in games. Nash was awarded the 1994 Nobel Prize in Economics for his achievements in game theory. Though Nash struggled with mental health issues, he continued his research. His life was loosely portrayed in the movie *Beautiful Mind*, an inspiring and encouraging story about the strength and beauty of the mind.

When there is no reason for any single player in a game to change his strategy, we call it a Nash equilibrium. In our payoff table, the Betray/Betray cell is the Nash equilibrium. So, why do we need a special name to describe this strange situation? Will we ever face this dilemma in real life? Yes, we will, because prisoners and non-prisoners alike play games like this! We play it...

...in economy: business trust game

Let's say two imaginary companies, A and B, are in the same industry. If A and B come to an agreement and set high prices, they each receive a profit of $700,000. However, if A breaks the agreement by lowering prices, A captures the market and gains an even greater profit of $900,000 while B's profit decreases to $100,000.

However, if each company tries to reduce its own risk and outmaneuver the other, both will lower prices and achieve a profit of $300,000 each. This is the Nash equilibrium.

		B: High price	B: Low price
A	High price	7, 7	1, 9
A	Low price	9, 1	3, 3

So, as in the Prisoner's Dilemma, it is a matter of trust. If businesses trust each other, they can potentially team up. But this is exactly what the government keeps an eye on! In the United States, antitrust laws prevent companies from coordinating prices. Set prices (and wages) disrupt the competitive free market. So, trust and cooperation are not necessarily "good," and for businesses, are sometimes illegal.

...just to show off: the game of chicken

Two players are driving toward each other in a video game. They can either keep driving straight or swerve away. If neither swerves, each player gets the highest penalty for the collision: (-1000, -1000). If both drivers swerve, they are both safe and the payoff is (0, 0). If one driver swerves while the other keeps going straight, the player who goes straight gets to yell "Chicken!" and gets +1. The driver who swerves gets -1.

The dilemma is that no one wants to be called chicken, but both drivers want to avoid a collision because a collision is the worst possible outcome.

Study the payoff table for the game of chicken.

What is the Nash equilibrium for the game of chicken?

... in politics

Just as in the game of chicken, when a country engages in an arms race, it builds its military to become stronger than its opponents.

		2nd driver's choice: Drive straight	2nd driver's choice: Swerve
1st driver's choice	Drive straight	-1000, -1000	1, -1
1st driver's choice	Swerve	-1, 1	0, 0

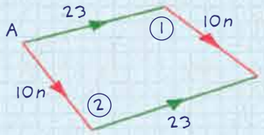
...in everyday life: traffic jam game

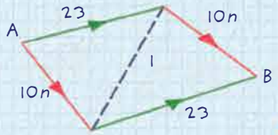
Drivers can take route 1 or route 2 to get from point A to point B. And drivers can travel either green road in 23 minutes and either red road in 10 × *n* minutes where *n* is the number of cars on the road at that time. Let's assume that the two dtrivers have agreed to use different routes.

Then the mayor builds a super-speedy highway to improve traffic. It takes only 1 minute to drive the entire length of the new highway.

Now every driver has to decide whether to use the new highway chasing the red roads, or keep the initial strategy. Fill in the payoff table to show how long it would take each driver to get from A to B.

		2nd driver's choice: Use highway	2nd driver's choice: Don't use
1st driver's choice	Use highway		
1st driver's choice	Don't use		





...in evolutionary biology: stag hunt game

Wolves can either cooperate and hunt as a pack or hunt alone.

If all wolves cooperate (hunt in a pack), the pack is more likely to catch larger prey. If only one wolf defects (hunts alone), it might have a decent individual payoff, but the pack suffers. If every wolf hunts alone, each may catch prey, but the pack's chances of long-term success are affected.

The most successful strategy for the pack is a balance where most wolves cooperate to maximize overall hunting success, which ensures the pack survives and stays healthy.

Scientists use the Prisoner's Dilemma to study cooperation and competition among animals. Why do some animals help each other while others do not? Scientists want to understand the impact on the group's survival.

...in health care: antibiotic prescription game

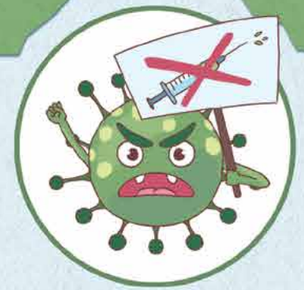
A doctor must decide whether to prescribe antibiotics.

Each doctor wants her patient to be healthy, so she should prescribe antibiotics. But this increases the population's resistance to the antibiotic and as doses go up, so do side effects. Eventually, the effect of the antibiotic is a net zero for everyone.

In this "game," the doctor who does not prescribe antibiotics "cooperates." The cooperator's payoff is negative, as if the medication did not exist. The payoff for the doctor who does prescribe the antibiotic (the "defector") uniformly decreases with each additional prescription.

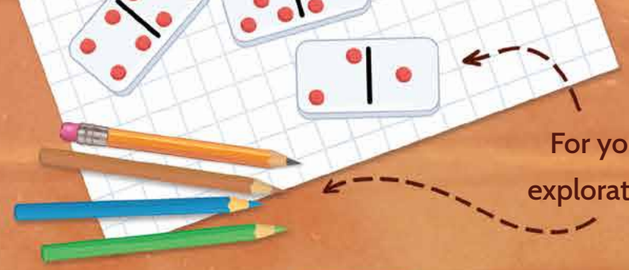
If a doctor chooses a less aggressive strategy (short of avoiding antibiotics completely) they all benefit more. However, as each doctor focuses on her individual patients, the likelihood of a worse outcome for the overall population increases.

The decision to get vaccinated is a dilemma. If the entire community gets vaccinated, there is a collective benefit of "herd immunity." However, an individual might choose not to get vaccinated to personally avoid side effects.

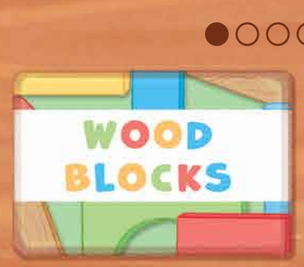


Game Theory was developed less than a hundred years ago. Yet people realized the impact of individual decision making on society long before that. Daniel Bernoulli commented in a paper he presented to the French Academy of Sciences in 1766, "I simply wish that, in a matter which so closely concerns the wellbeing of the human race, no decision shall be made without all the knowledge which a little analysis and calculation can provide."

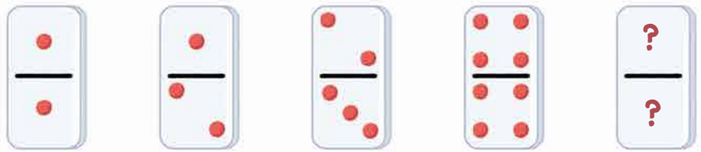
MTWIST

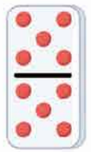


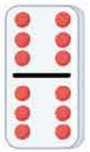
For your mathematical exploration, you will need:



Which domino should be fifth in the row?



A 

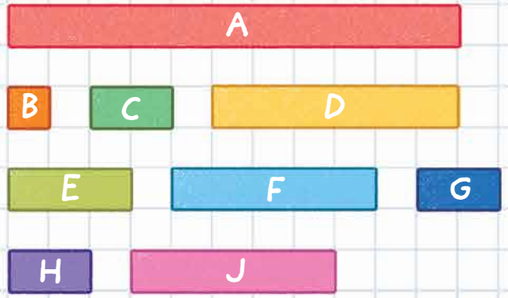
B 

C 

D 

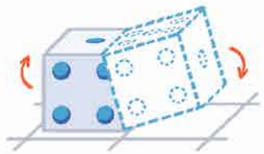
Follow the rules below to build a tower.

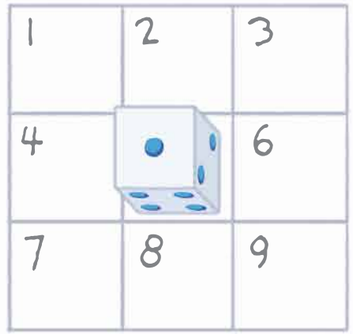
1. Block A rests on blocks B, C, and J.
2. Block B rests on block D.
3. Block C rests on block F.
4. Block D rests on blocks E and F.
5. Block F rests on blocks G and H.



Rotate the die six times and get it to square 7 with 6 dots face up.

For example, with one move right, the die will be on square 6 with 5 dots face up.

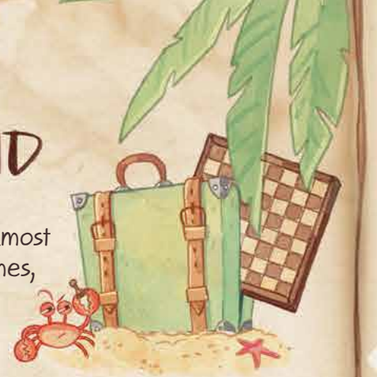




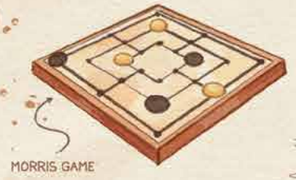
Play with a friend! Roll two dice, add the two numbers that you roll, and find and mark your number on the board. If your number appears more than once, choose the circle that is better. If all the circles with your number are already marked, roll the dice again. Your goal is to mark four circles in a row, either horizontally, vertically, or diagonally.

8	2	7	12	3	8	5
4	10	5	6	9	10	7
7	3	8	10	5	6	4
9	11	4	2	6	7	9
6	5	7	8	3	5	6
8	7	4	11	9	3	12

FROM A JOURNAL FOUND ON A DESOLATE ISLAND



9 March Went to the wreck, found the steersman's chest. It was broken and almost full of water and sand. I found nothing useful except for a set of games, including an old chessboard in a thick well-worn leather case. As I had learned not to despair of anything, I took all the games to my hut.



10 March I went a-fishing and caught enough fish for several days. So I decided to play chess against myself and I was surprised with the joy of it!

11–15 March Worked on the wreck, played chess.

16 March Rained all day, and I stayed within. But I could not play chess any more. Man cannot beat himself at this game because all his moves are immediately clear.

17 March I resolved that I would build me a wall around my camp and so I brought three great fir planks off from the decks. In the evening, I thought to myself What if I were to modify a chessboard? Then might it perhaps become a better game?

Problem 1. Other than squares, what regular polygons (all of the same size and shape) can tile a plane?

20 March How strange a chequer-work of Providence is the life of man! It was so violently hot that I could not leave my cave and I thought of nothing but my misery. In the meantime, my huge new chessboard is almost ready. Cannot determine how the knight and the bishop should move on the hexagonal board. Should I invent new pieces?

Problem 2. Can you tile a plane using regular polygons of two different types such as a triangle and a square? Find at least two possibilities.

21 March Traveled nearly four miles and found a lot of trees with different fruits, and I particularly rejoiced at the grapes upon the vines. Thought a lot about my new chessboard.

22 March Brought more planks. Today I decided that I could also use irregular polygons on my board. Is not regularity merely imposed upon us by society? I quieted my mind with this and started with triangles again.

Problem 3. Can you tile a plane with irregular triangles? (All the tiles should be the same shape and size.)

23 March I hung grapes to dry in the sun. The sweet of raisins will be so delicious! Spent the evening pondering how many bishops should be used on a chessboard made of triangles. I could paint the board in two colours or in three instead. Would three bishops make it a better game? Should I return to tiled quadrilaterals?

Problem 4. Can you tile a plane with arbitrary quadrilaterals? (All the tiles should be the same shape and size.) What if the quadrilaterals are concave?

26 March Sorted through my winter store of raisins that are drying in the sun. Although they are rather round, I can lay them almost flush together. Could that be a hint for my game?

Problem 5. Can you tile a plane with arbitrary dodecagons of the same shape and size? Can you do it in such a way that no point on the plane belongs to more than three dodecagons?

1 April Today I traveled even farther inland and found a cove and a human skull next to the cove. I am not the first human creature to set foot upon this ground!

2–5 April Worked on the map of my island.

6 April It is next to miraculous! I planned to use my aged leather case that contained the chess set. As I turned it inside out, I found a barely noticeable drawing that resembles my map. It is badly worn and some details are missing, but I know that it is a map of my island because it has a cove and a grape grove. And a chest!

Problem 6. My map of the island is on the left. Can you mark the spot where the treasure chest should be? Use the worn and distorted map on the right to mark the location of the chest on the left. The grove and the cove have not moved.

7 April But the next thing was a shovel or spade; this was so absolutely necessary, that indeed I could do nothing effectually without it; but what kind of one to make I knew not. So I went to the woods and, with great labour, cut a piece of wood that in the Brazils they call the iron tree.

14 April I worked that piece by little and little into the form of a shovel. Tomorrow will be the day!

This journal comes to an end at this point. The contents of the chest remain a significant mystery. Our sincere thanks are extended to Daniel Defoe for his generous assistance in the development of this text.

Daniel Defoe, *The Life and Adventures of Robinson Crusoe*. London: Thomas Nelson and Sons, 1874.

ANSWERS

Scalify-Magnify (p. 6)

1) 3 large > 20 small
2) 3 tubes
3) about 1.3 kg or 2.8 lb
4) 10 times

Missing Pieces (p. 10)

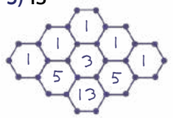
Hint: Think about how many marbles you need to do it.

Solution: Suppose it is possible. Let's order the piles by the number of marbles they contain. Then the first pile has at least 1 marble, the second pile has at least 2 marbles, and so on. That makes the total number of marbles at least $1 + 2 + 3 + \cdots + 9 = 45$, but you only have 44 marbles.

So, it is not possible.

MChallenge (p. 12)

1) 1034687
2) 26
3) 126 ways
4) 36
5) 13



6) 2
7) 7 squares
8) Wednesday

Beat the Offbeat Ogre (p. 14)

With a d12, you have a $\frac{6}{12} = \frac{1}{2}$ chance of rolling a 7 or up.

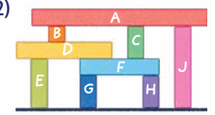
With 2d6, you have a $\frac{21}{36} = \frac{7}{12}$ chance, which is greater than a $\frac{1}{2}$ chance of rolling a 7 or up.

Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

MTwist (p. 20)

1) B

2)



3)

Game of Trust (p. 16)

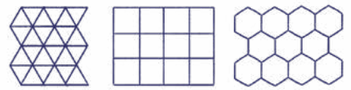
Game of Chicken: (-1, 1) and (1, -1)
One driver swerves, the other does not, and there is no reason for either to change his strategy.

Traffic jam game: No driver will benefit from an individual refusing to drive on the highway – and the trip time will increase for all traffic members.

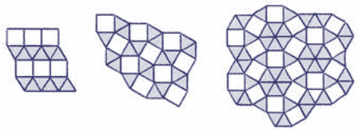
	Use hwy	Don't use hwy
Use hwy	41, 41	31, 43
Don't use hwy	43, 31	33, 33

From a Journal Found on a Desolate Island (p. 22)

1)



2)



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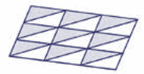
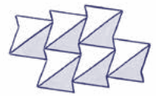
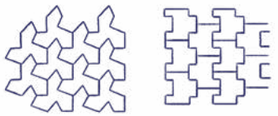


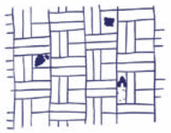
4)



5)



6)



Harvard-MIT Math Tournament (HMMT) 2023

Out of 137 Teams, RSM Placed:

4th in the Team Round

11th in the Guts Round & the Sweepstakes Round

RSM Team A:

Luca Pieleanu, Rajarshi Mandal, Jefferson Ji,
Kristiyan Kurtev, Arthur Liangchen Li, Andrew Kalashnikov

RSM
Russian School of Mathematics